

Homework 6, Spring 2023:

Problem 6.1: Construct a Schwartz function Ψ that satisfies $\sum_{j \in \mathbb{Z}} |\widehat{\Psi}(2^{-j}\xi)|^2 = 1$ for all $\xi \in \mathbb{R}^n \setminus \{0\}$ and whose Fourier transform is supported in the annulus $6/7 \leq |\xi| \leq 2$ and is equal to 1 on the annulus $1 \leq |\xi| \leq 13/7$.

Problem 6.2: Suppose that $\phi(\xi)$ is a smooth function on \mathbb{R}^n that vanishes in a neighborhood of the origin and is equal to 1 in a neighborhood of infinity. Prove that the function $e^{i\xi_j|\xi|^{-1}}\phi(\xi)$ is in $\mathcal{M}_p(\mathbb{R}^n)$ for $1 < p < \infty$.