

Homework 9:

Problem 9.1: Suppose that $\phi : \mathbb{R} \rightarrow \mathbb{R}$ satisfies that

$$\phi(x_0) = \phi'(x_0) = \phi''(x_0) = 0,$$

while $\phi'''(x_0) \neq 0$. If $\psi \in C_c^\infty(\mathbb{R})$ is supported in a sufficiently small neighborhood of x_0 , prove that

$$\int_{\mathbb{R}} e^{i\lambda\phi(x)} \psi(x) \, dx = \lambda^{-1/3} \sum_{j=0}^N a_j \lambda^{-j/3} + O(\lambda^{-(N+2)/3})$$

for all $\lambda > 1$ and nonnegative integer N .