Homework 7:

Problem 7.1: If

$$\|\widehat{f\mathrm{d}\sigma}\|_{L^{p'}(\mathbb{R}^n)} \lesssim \|f\|_{L^{q'}(S^{n-1})}$$

holds for all $f \in L^{q'}(S^{n-1})$, then

$$q \leq \frac{n-1}{n+1}p'.$$

Problem 7.2: Suppose that $S$ is a bounded subset of a hyperplane in $\mathbb{R}^n$. Prove that if $\|\hat{f}|_S\|_{L^1(S)} \leq C\|f\|_{L^p(\mathbb{R}^n)}$ for all $f \in \mathcal{S}(\mathbb{R}^n)$ then necessarily $p = 1$, in other words, there cannot be a nontrivial restriction theorem for flat (affine) surfaces.