Homework 6:

Problem 6.1: Suppose that $\phi : \mathbb{R} \to \mathbb{R}$ satisfies that
$$\phi(x_0) = \phi'(x_0) = \phi''(x_0) = 0,$$
while $\phi'''(x_0) \neq 0$. If $\psi \in C_c^\infty(\mathbb{R})$ is supported in a sufficiently small neighborhood of x_0, prove that

$$\int_{\mathbb{R}} e^{i\lambda\phi(x)}\psi(x)\,\mathrm{d}x = \lambda^{-1/3}\sum_{j=0}^{N} a_j \lambda^{-j/3} + O(\lambda^{-(N+2)/3})$$

for all $\lambda > 1$ and nonegative integer N.