Homework 5:

Problem 5.1: Fix a nonzero Schwartz function h on the line whose Fourier transform is supported in the interval $[-1/8, 1/8]$. For $\{a_j\}$ a sequence of numbers, set

$$f(x) = \sum_{j=1}^{\infty} a_j e^{2\pi i 2^j x} h(x).$$

Prove that for all $1 < p < \infty$ there exists a constant C_p such that

$$\|f\|_{L^p(\mathbb{R})} \leq C_p (\sum_j |a_j|^2)^{1/2} \|h\|_{L^p}.$$

Problem 5.2: (The exercise on P.27 Chapter 4, Wolff) Using translation and multiplication by characters, construct a sequence of Schwartz functions $\{\phi_n\}$ so that

(1) Each $\{\phi_n\}$ has the same L^p norm.
(2) Each $\widehat{\phi_n}$ has the same $L^{p'}$ norm.
(3) The supports of the $\widehat{\phi_n}$ are disjoint.
(4) The supports of the $\{\phi_n\}$ are "essentially disjoint" meaning that

$$\|\sum_{n=1}^{N} \phi_n\|_p^p \approx \sum_{n=1}^{N} \|\phi_n\|_p^p \approx N$$

uniformly in N.